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Using Standard Division to Solve Multi-Objective Quadratic Fractional Programming Problems

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Abstract

In this paper, Standard Division (SD) technique is used to solve multi-objective Quadratic fractional programming problems (MOQFPP), each objective functions has the following form $max.(min.)z = \frac{(a_1^t x+\alpha)(a_2^t x+\beta)}{(b_1^t x+\gamma)(b_2^t x+\delta)}$, and an algorithm suggested for it. Also the problems solved by Chandra Sen technique. We test several examples but introduce two of them in this paper and compare the objective functions between SD and Sen's technique which is indicate SD technique batter than the other.

Introduction

Sharma and Jitendra (2013) [2] introduce new approach for linear factorized quadratic optimization and a quadratic fractional optimization problem. Sulaiman and Nawkhass (2013) [3] studied and suggested transforming and solving multi-objective quadratic fractional programming problems. In (2013) [4] Sulaiman and Nawkhass approach a new modified simplex method to solve quadratic fractional programming problem and compared it to a traditional simplex method by using pseudoaffinity of quadratic fractional functions. Hasan and Babul (2014) [1] studied and suggested transforming an alternative method for solving quadratic fractional programming problems with homogenous constraints. An SDP approach for solving quadratic fractional programming problems is studied by Nguyen, Sheu, Yong Xia (2014) [5]
In order to extend this work we have defined a MOQFPP and investigated the algorithm to solve multi-objective quadratic fractional function by using SD technique, irrespective of the number of objectives with less computational burden and also solved the problems by Chandra Sen technique. Finally different a technique was compared which is show in tables 3.

Multi-Objective Quadratic Fractional Programming Problem

Multi-Objective functions that are the ratio of quadratics functions are said to be MOQFPP which is defined as:

$$\left.\begin{array}{l} Max.Z_1 = (a_{11}^t x+\alpha_1)(a_{21}^t x+\beta_1)/(b_{11}^t x+\gamma_1)(b_{21}^t x+\delta_1) \\ Max.Z_2 = (a_{12}^t x+\alpha_2)(a_{22}^t x+\beta_2)/(b_{12}^t x+\gamma_2)(b_{22}^t x+\delta_2) \\ \quad\vdots \\ Max.Z_r = (a_{1r}^t x+\alpha_r)(a_{2r}^t x+\beta_r)/(b_{1r}^t x+\gamma_r)(b_{2r}^t x+\delta_r) \\ Min.Z_{r+1} = (a_{1r+1}^t x+\alpha_{r+1})(a_{2r+1}^t x+\beta_{r+1})/(b_{1r+1}^t x+\gamma_r)(b_{2r+1}^t x+\delta_{r+1}) \\ \quad\vdots \\ Min.Z_s = (a_{1s}^t x+\alpha_s)(a_{2s}^t x+\beta_s)/(b_{1s}^t x+\gamma_s)(b_{2s}^t x+\delta_s) \end{array}\right] \quad (1)$$

Subject to:

$$Ax = b \quad (2)$$

$$x \geq 0 \quad (3)$$

Where b is $m-$dimensional vector of constants, x is $n-$dimensional vector of decision variables and A is a $m \times n$ matrix of constants,a_{1i}, a_{2i} , b_{1i} , b_{2i} are n-dimensional vector of constants, where and $\alpha_{1i}, \beta_{1i}, \gamma_{1i}, \delta_{1i}$ are scalars constants, where $i = 1,2,3, \dots, s$.

Sen's Technique

The same approach which was taken by Sen (1983) [6] is allow us in this work to formulate the constrained objective functions is given in equation (1). Suppose we obtain a single value corresponding to each of the objective functions of the MOQFPP of formula (1) by use new approach for linear factorized quadratic optimization [2], being optimized individually subject to the constraints (2) and (3) as follows:

$$\left.\begin{array}{l} Max.Z_1 = \varphi_1 \\ Max.Z_2 = \varphi_2 \\ \quad\vdots \\ Max.Z_r = \varphi_r \\ Min.Z_{r+1} = \varphi_{r+1} \\ \quad\vdots \\ Min.Z_s = \varphi_s \end{array}\right] \quad (4)$$

Where $\varphi_1, \varphi_2, \dots, \varphi_s$are values of the objective functions, the level of the decision variables may not necessarily be the same for all optimal solutions in presence of conflicts among objectives. In this study the common set of decision variables to be the best compromising optimal solution that we can determine for the common set of the decision variables from the following combined objective function, which formulate the MOLFPP given in following equation.

$$Max.Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|} = \quad (5)$$

where $\varphi_i \neq 0, i = 1,2,\dots,s$. Subject to constraints (2) and (3), the optimum value of the objective functions$\varphi_i, i = 1,2,\dots,s$ may be positive or negative. and Z_i represents the $Max.z$ objective functions (when $i = 1,\dots,r$) and (when $i = r+1,\dots,s$) represents the $Min.z$ objective function in (1). Finally the formula (5) was solved with same constraint (2) and (3).

Standard Division Technique

Here propose an idea for using standard Division technique to solve multiple objective programming based on the idea that Formulating the combined objective function (6) as the formula (5) to determine the common set of decision variables, to solving the MOQFPP.

$$Max. Z = \sum_{i=1}^{r} \frac{Z_i}{SD} - \sum_{i=r+1}^{s} \frac{Z_i}{SD} = \frac{SM-SN}{SD} \qquad (6)$$

Subject to constraint (2) and (3).

Where $SD = \sqrt{\frac{\sum_{i=1}^{s}(\varphi_i-\bar{\varphi})^2}{n}}$, $\bar{\varphi} = \frac{\sum_{i=1}^{s}\varphi_i}{n}$

$SM = \sum_{i=1}^{r} Z_i, SN = \sum_{i=r+1}^{s} Z_i.$

SM:is summation of maximum objective functions , SN:is summation of minimum objective functions

The Algorithm of Standard Division Technique to Solve MOQFPP

To obtaining the optimal solution for the MOQFPP in formula (6), an algorithm is defined which can be written as:

Step1: Give arbitrary value to each objective functions which are to be maximized or minimized or both.

Step2: Solve the all objective functions by use new approach for linear factorized quadratic optimization.

Step3: Convert the maximum objectives to single objective function, let be *SM*. Convert the minimum objectives to single objective function, let be *SN*.

Step4: Combined objective functions by using formula (6)

Step5: Optimize the combined objective function which is subject to (2) and (3).

Numerical Examples:

Example 1:-

$$Max. Z_1 = (x_1 + 3x_2 + 1)(4x_1 + 2x_2 + 2)/(3x_1 + 3x_2 + 3)(2x_1 + 2x_2 + 2)$$

$$Max. Z_2 = (2x_1 + 6x_2 + 2)(12x_1 + 6x_2 + 6)/(5x_1 + 5x_2 + 5)(x_1 + x_2 + 1)$$

$$Max. Z_3 = (4x_1 + 12x_2 + 4)(20x_1 + 10x_2 + 10)/(4x_1 + 4x_2 + 4)(2x_1 + 2x_2 + 2)$$

$$Min. Z_4 = (-6x_1 - 8x_2 - 6)(16x_1 + 8x_2 + 8)/(3x_1 + 3x_2 + 3)(4x_1 + 4x_2 + 4)$$

$$Min. Z_5 = (5x_1 + 15x_2 + 5)(-8x_1 - 4x_2 - 4)/(6x_1 + 6x_2 + 6)(x_1 + x_2 + 1)$$

$$Min. Z_6(-3x_1 - 9x_2 - 3)(24x_1 + 12x_2 + 12)/(2x_1 + 2x_2 + 2)(5x_1 + 5x_2 + 5)$$

Subject to

$$2x_1 + x_2 \leq 4$$

$$5x_1 + 2x_2 \leq 25$$

$$x_1, x_2 \geq 0$$

Solution

First solve each objective functions by use new approach for linear factorized quadratic optimization [2] and the values of each objective functions present in following table.

Table-1: Value Of objective Function of Example 1.

i	x_i	φ_i
1	(0,4)	0.8
2	(0,4)	6.2
3	(0,4)	13
4	(0,4)	-10.4
5	(0,4)	-8.6
6	(0,4)	-9.3

Now

$$SM = \sum_{i=1}^{3} Z_i = \frac{(32x_1 + 96x_2 + 32)(116x_1 + 58x_2 + 58)}{(6x_1 + 6x_2 + 6)(30x_1 + 30x_2 + 30)}$$

$$SN = \sum_{i=4}^{6} Z_i = \frac{(-96x_1 - 288x_2 - 96)(268x_1 + 134x_2 + 134)}{(16x_1 + 16x_2 + 16)(45x_1 + 45x_2 + 45)}$$

A:-using*SD technique*

By apply formula (6) we get

$$Max. Z = \frac{SM-SN}{SD} = \frac{(8x_1+24x_2+8)(32x_1+16x_2+16)}{(5x_1+5x_2+5)(8.831x_1+8.831x_2+8.831)}$$

Subject to

$$2x_1 + x_2 \leq 4$$

$$5x_1 + 2x_2 \leq 25$$

$$x_1, x_2 \geq 0$$

By solving the problem we get $Max. Z = 6.28$ at the optimal point $x_1 = 0, x_2 = 4$.

B:*-using Chandra Sen technique*

By apply formula (5) we get

$$Max. Z = \frac{SM-SN}{|\varphi_i|} = \frac{(16x_1+28x_2+16)(16x_1+8x_2+8)}{(5x_1+5x_2+5)(56.488x_1+56.488x_2+56.488)}$$

Subject to

$$2x_1 + x_2 \leq 4$$

$$5x_1 + 2x_2 \le 25$$

$$x_1, x_2 \ge 0$$

By solving the problem we get $Max.Z = 0.9819$ at the optimal point $x_1 = 0, x_2 = 4$.

Example 2:-

$$Max.Z_1 = (3x_1 + 2x_2 + 4)(2x_1 + 5x_2 + 3)/(2x_1 + 2x_2 + 2)(x_1 + x_2 + 1)$$

$$Max.Z_2 = (6x_1 + 4x_2 + 8)(6x_1 + 15x_2 + 9)/(3x_1 + 3x_2 + 3)(5x_1 + 5x_2 + 5)$$

$$Max.Z_3 = (12x_1 + 8x_2 + 16)(4x_1 + 10x_2 + 6)/(x_1 + x_2 + 1)(4x_1 + 4x_2 + 4)$$

$$Min.Z_4 = (9x_1 + 6x_2 + 12)(-8x_1 - 20x_2 - 12)/(4x_1 + 4x_2 + 4)(2x_1 + 2x_2 + 2)$$

$$Min.Z_5 = (-15x_1 + 10x_2 - 20)(12x_1 + 30x_2 + 18)/(3x_1 + 3x_2 + 3)(6x_1 + 6x_2 + 6)$$

Subject to

$$6x_1 + 5x_2 \le 30$$

$$2x_1 + x_2 \le 8$$

$$5x_1 + 9x_2 \le 45$$

$$x_1, x_2 \ge 0$$

Solution

First solve each objective functions by use new approach for linear factorized quadratic optimization [2] and the values of each objective functions present in following table.

Table-2: Value Of objective Function of Example 2.

i	x_i	φ_i
1	*(0,5)*	*5.44*
2	*(0,5)*	*4.35*
3	*(0,5)*	*21.7*
4	*(0,5)*	*-16.3*
5	*(0,5)*	*-18.1*

Now

$$SM = \sum_{i=1}^{3} Z_i = \frac{(9x_1 + 6x_2 + 12)(19.2x_1 + 48x_2 + 28.8)}{(2x_1 + 2x_2 + 2)(5x_1 + 5x_2 + 5)}$$

$$SN = \sum_{i=4}^{5} Z_i = \frac{(-11.4x_1 - 7.6x_2 - 15.2)(10x_1 + 25x_2 + 15)}{(2x_1 + 2x_2 + 2)(3x_1 + 3x_2 + 3)}$$

A:-using*SD technique*

By apply formula (6) we get

$Max.Z = \frac{SM-SN}{SD} = \frac{(6x_1+4x_2+8)(2x_1+5x_2+3)}{(x_1+x_2+1)(5x_1+5x_2+5)}$

Subject to

$$6x_1 + 5x_2 \leq 30$$

$$2x_1 + x_2 \leq 8$$

$$5x_1 + 9x_2 \leq 45$$

$$x_1, x_2 \geq 0$$

By solving the problem we get $Max.Z = 4.3556$ at the optimal point $x_1 = 0, x_2 = 5$.

B:-*using Chandra Sen technique*

By apply formula (5) we get

$Max.Z = \frac{SM-SN}{|\varphi_i|} = \frac{(36x_1+24x_2+48)(15.16x_1+37.9x_2+22.74)}{(11x_1+11x_2+11)(90x_1+90x_2+90)}$

Subject to

$$6x_1 + 5x_2 \leq 30$$

$$2x_1 + x_2 \leq 8$$

$$5x_1 + 9x_2 \leq 45$$

$$x_1, x_2 \geq 0$$

By solving the problem we get $Max.Z = 1.0005$ at the optimal point $x_1 = 0, x_2 = 5$.

Results and Discussion

Depending on table shown below, where Standard division, Sen's technique is applied to solve MOQFPP of the numerical Examples 1 and 2. The comparisons of these methods are based onthe value of the objective function.

Table-3: Comparison between Standard Division, Sen's technique:

Methods	*Example 1*		*Example 2*	
	$Max.z$	(x_1, x_2)	$Max.z$	(x_1, x_2)
Sen's Technique	0.9819	*(0,4)*	1.0005	*(0,5)*
Standard Division	*6.28*	*(0,4)*	4.3556	*(0,5)*

Standard Division is better than Sen's technique, According to the obtained results.

Conclusions

This work, use two techniques (Standard Division, Sen's) to solve multi-objective quadratic fractional programming problems (MOQFPP), We also developed an algorithm to find optimal solution, the comparisons of these methods are based on the value of the objective function as indicate in Table 3, we found Standard Division techniqueis better than Sen’s technique.

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